Interferometric measurement of refractive indices

K. Betzler, A. Gröne, N. Schmidt, and P. Voigt

Universität Osnabrück, Fachbereich Physik, Postfach 4469, D 4500 Osnabrück,
Federal Republic of Germany

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A simple method for the measurement of the refractive indices of parallel plate samples is described that uses the shift of the interference pattern when rotating the sample in one arm of a Mach–Zehnder interferometer. The theoretical fringe pattern shift is calculated and the accuracy of the method is derived. A typical accuracy of about 10^{-4} can be achieved at low refractive indices. The method is especially useful when—because of other reasons—the samples are provided as plates or when prisms are difficult to fabricate.

The flatness, parallelism, and homogeneity of crystal plates are often characterized by interferometry. Small relative shifts of the index of refraction can be measured interferometrically as well.[1,2] For these measurements usually Mach–Zehnder-type interferometers are used. We describe a method to use a Mach–Zehnder interferometer (MZI) for the determination of the absolute index of refraction. Compared to the standard prism method the interferometric method yields the opportunity to use samples with plate geometry. Such samples are often necessary for the measurement of other physical properties and, from many crystals, can be fabricated easier than prisms.

The experimental setup for the measurements is sketched in Fig. 1. To a standard Mach–Zehnder interferometer a computer-controlled rotation table and an image sensing system are added. In the simplest case the image sensing system may consist of a photodiode to monitor a definite spot in the interference pattern and the appropriate electronics—amplifier and analog-to-digital converter. Intensity evaluation (peak detection, etc.) then is due to the computer software. Superior to such a simple system is a direction sensitive detection system—that means a system which both counts fringes and detects the walking direction of the fringe pattern. Such a detection system consists of at least two photodiodes adjacent to each other or, better, a quadrant photodiode.

The refractive index of the sample is determined by measuring the shift $m(\varphi)$ of its fringe pattern as a function of the rotation angle. A simple evaluation of this function is achieved when the MZI is tuned in such way that the interference fringes show up perpendicular to the rotation axis of the crystal. In this case the theoretical fringe pattern shift $m_t(\varphi)$ when rotating the crystal is given by

$$m_t(\varphi) = \frac{d}{\lambda}\left(\frac{n - \cos(\varphi - \varphi')}{\cos\varphi'} - n + 1\right), \qquad (1)$$

$$\varphi' = \arcsin\,(\sin\varphi/n)\,, \qquad (2)$$

with d: crystal thickness, λ: laser wavelength, φ: rotation angle, n: index of refraction, and φ': beam direction inside the crystal measured from normal incidence.

The theoretical fringe shift functions for several values of the index of refraction are depicted in Fig. 2. In the calculation a crystal thickness of $d = 10$ mm and a laser wavelength of $\lambda = 632.8$ nm were assumed.

Equations (1) and (2) are derived for optically isotropic samples; with their geometrical arrangement the crystal symmetry need not be taken into consideration. Measurements on optically uniaxial and biaxial samples, however, have to be carried out in a special way to keep the evaluation simple:

(a) Uniaxial case: The sample must be prepared with its slab geometry and its rotation axis lying parallel to the optical axis. Light polarization parallel to the rotation axis then yields the extraordinary index of refraction n_e, light polarization perpendicular to the rotation axis in turn the ordinary index n_o. In Eqs. (1) and (2) n has to be replaced by n_e or n_o, respectively.

(b) Biaxial case: Optically biaxial crystals are characterized by three different refractive indices. To measure all of them, at least two different geometries must be chosen. The sample plate must be fabricated with the used optical planes lying perpendicular to one of the axes of the indicatrix. The rotation axes for the two geometries are then chosen parallel to the two remaining axes of the indicatrix. Polarizing parallel to the rotation axis yields the respective index of refraction—two of the indices (n_1 and n_2) can be measured in this way. To measure the third index n_3 either an additional sample with the appropriate configuration has to be used or a measurement in one of the two other configurations with polarization perpendicular to the rotation axis has to be evaluated. Equations (1) and (2) change for that case: n has to be replaced by an angle-dependent effective index $n_{\text{eff}}(\varphi')$ given by

$$n_{\text{eff}}(\varphi') = (n_1^{-2}\cos^2\varphi' + n_3^{-2}\sin^2\varphi')^{-1/2}\,. \qquad (3)$$

The numerical evaluation of the measured dependence

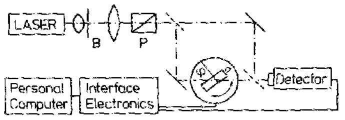

FIG. 1. Experimental setup for refractive index measurements with a Mach–Zehnder interferometer (B: beam expander, P: polarizer).

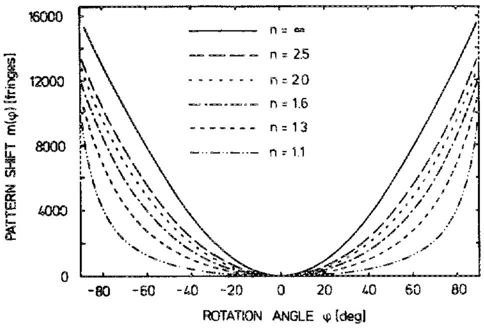

FIG. 2. Calculated interference-pattern shift which is caused by the rotation of plate-shaped samples (thickness 10 mm) in a Mach–Zehnder interferometer. Curves for different refractive indices n are given. The pattern shift $m(\varphi)$ is the number of fringes to be counted when rotating the sample.

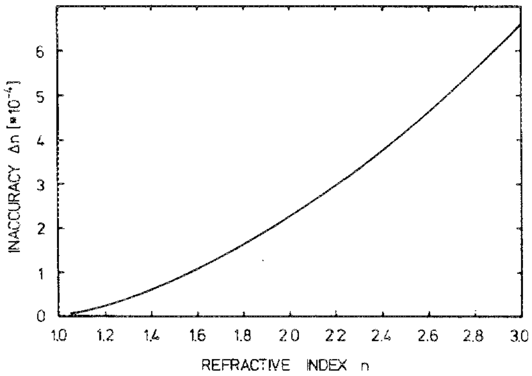

FIG. 3. Calculated inaccuracy Δn as a function of the refractive index.

$m(\varphi)$ is done in two steps: First the origin of the φ axis is shifted until the curve is exactly symmetric, then a numerical fit of the theoretical to the experimental curve is done. As the symmetrization has been done separately, in the fitting procedure the refractive index n is used as the only free parameter and thus can be easily determined by iterative linear interpolation which starts at two appropriate values for n. For uni- and biaxial crystals Eqs. (1) and (2) have to be changed in the discussed way and, for n_3, Eq. (3) has to be included in the iteration scheme.

In order to get a rough estimate for the method's accuracy we calculated the maximum error in the refractive index Δn as a function of n. For the calculation, the following assumptions were taken: crystal thickness $d = 10$ mm with an uncertainty of $\Delta d = \pm 10^{-3}$ mm, overall effective error of the fringe shift fitting procedure corresponding to an error of ± 0.1 fringe when measuring the fringe shift between two angles separated by 75°. This or an even better accuracy is achieved when all of the measured fringe maxima are included in the fitting procedure and when the "noise" in the rotation table is strongly random. The uncertainty in the laser wavelength was assumed to be negligible compared to the other errors. The inaccuracy Δn calculated in that way is shown in Fig. 3. At low refractive indices an uncertainty of less than 10^{-4} in n can be achieved, up to higher n the error rises considerably, a drawback shared with other methods of index measuring.

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Improved cavity alignment system for Spectra Physics 380 D single-mode ring dye lasers

A. M. van der Spek, P. Magendans, and A. H. Kemper

Department of Physics, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

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A description and working experience of an improved mechanical alignment system of a Spectra Physics 380 D ring dye laser is given.

Aligning the cavity of a ring dye laser is known to be a cumbersome and sometimes tedious procedure. The procedure for a completely dealigned cavity starts with an open cavity, i.e., a cavity from which all elements have been removed. By successively adding the galvoplates, the astigmatism compensator, the unidirectional device, the birefringent filter, and the thick and thin étalons, single-mode operation at the desired wavelength can be obtained. This procedure may take several hours up to a day. Moreover, previous experience has shown that an aligned cavity may become dealigned overnight. This may be due to hysteresis in the three-point cavity mirror mounts, and to tolerances in the adjustment screws. This has led us to design and construct a new mechanical alignment system. Design goals were: (1) All 20 new adjustment points should be located in the original place. (2) The adjustment screws should be tolerance free.